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LETTER TO THE EDITOR

$\Delta\Delta$ and $\Delta\Delta\Delta$ bound states

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Abstract

We calculate the two- and three-body spectra of deltas using a chiral quark cluster model and a meson-exchange model for the $\Delta\Delta$ interaction. The ordering of the states is pretty much model independent. Both models predict the existence of four $\Delta\Delta$ bound states that couple to the NN system. Three of these states can be identified with known NN states. The fourth state corresponds to a new NN resonance with isospin 0, spin 3 and positive parity. A possible signal of this resonance appears in recent analyses of NN data.

Recently, a $\Delta\Delta$ interaction derived from a chiral quark cluster model has been used to calculate the $\Delta\Delta$ and $\Delta\Delta\Delta$ bound-state spectra [1]. The method was later generalized to also include the $N\Delta$, $NN\Delta$ and $N\Delta\Delta$ spectra [2, 3]. Traditionally, the two-baryon interaction has been described in terms of meson-exchange potentials. In this paper we are going to reanalyse the $\Delta\Delta$ and $\Delta\Delta\Delta$ systems using both kinds of models. We would like to point out that a new resonance is predicted by our calculations. The same $\Delta\Delta$ bound state has also been predicted by other models [4–8]. In particular, Goldman and his collaborators [5, 6] have called this state the deltaron. In a recent paper [9], Wong has proposed a method to search for this state experimentally. Finally, we analyse the special role played by quark Pauli blocking effects on the two- and three-body spectra.

The $\Delta\Delta$ interaction generated by the chiral quark cluster model has been described in detail in [1]. A meson-exchange model for the $N\Delta$ interaction has been constructed in [10, 11]. We will generalize this model for the case of the $\Delta\Delta$ interaction. The meson-exchange contributions to the $\Delta\Delta$ potential are not derived from field-theoretic Lagrangians describing the fundamental couplings of mesons to the Δ isobar. Instead, the Δ isobar is assumed to behave qualitatively like a nucleon, and known two-nucleon meson-exchange forms are simply transcribed to the Δ isobar, taking care of the difference in spin, isospin and coupling strengths. This potential includes the exchange of π, ρ, σ and ω mesons between the baryons. It has the form

$$V_{\Delta\Delta}(\vec{r}) = V_\pi(\vec{r}) + V_\sigma(\vec{r}) + V_\rho(\vec{r}) + V_\omega(\vec{r}) \quad (1)$$

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where the contributions of the different mesons are given by

$$V_\pi(\vec{r}) = \frac{1}{3}\frac{f_\pi^2}{4\pi}\frac{1}{25}m_\pi\left\{\left[Y(m_\pi r) - \frac{\Lambda_\pi^3}{m_\pi^3}Y(\Lambda_\pi r)\right]\vec{\sigma}_\Delta(1)\cdot\vec{\sigma}_\Delta(2) + \left[H(m_\pi r) - \frac{\Lambda_\pi^3}{m_\pi^3}H(\Lambda_\pi r)\right]S_{\Delta\Delta}(1,2)\right\}\vec{\tau}_\Delta(1)\cdot\vec{\tau}_\Delta(2) \tag{2}$$

$$V_\rho(\vec{r}) = \frac{1}{3}\frac{f_\rho^2}{4\pi}\frac{1}{25}m_\rho\left\{2\left[Y(m_\rho r) - \frac{\Lambda_\rho^3}{m_\rho^3}Y(\Lambda_\rho r)\right]\vec{\sigma}_\Delta(1)\cdot\vec{\sigma}_\Delta(2) - \left[H(m_\rho r) - \frac{\Lambda_\rho^3}{m_\rho^3}H(\Lambda_\rho r)\right]S_{\Delta\Delta}(1,2)\right\}\vec{\tau}_\Delta(1)\cdot\vec{\tau}_\Delta(2) + \frac{g_\rho^2}{4\pi}m_\rho\left[Y(m_\rho r) - \frac{\Lambda_\rho^3}{m_\rho^3}Y(\Lambda_\rho r)\right]\vec{\tau}_\Delta(1)\cdot\vec{\tau}_\Delta(2) \tag{3}$$

$$V_\sigma(\vec{r}) = -\frac{g_\sigma^2}{4\pi}\frac{1}{3}m_\sigma\left[Y(m_\sigma r) - \frac{\Lambda_\sigma^3}{m_\sigma^3}Y(\Lambda_\sigma r)\right] \tag{4}$$

$$V_\omega(\vec{r}) = \frac{g_\omega^2}{4\pi}m_\omega\left[Y(m_\omega r) - \frac{\Lambda_\omega^3}{m_\omega^3}Y(\Lambda_\omega r)\right] + \frac{1}{3}\frac{f_\omega^2}{4\pi}m_\omega\left\{2\left[Y(m_\omega r) - \frac{\Lambda_\omega^3}{m_\omega^3}Y(\Lambda_\omega r)\right]\vec{\sigma}_\Delta(1)\cdot\vec{\sigma}_\Delta(2) - \left[H(m_\omega r) - \frac{\Lambda_\omega^3}{m_\omega^3}H(\Lambda_\omega r)\right]S_{\Delta\Delta}(1,2)\right\}. \tag{5}$$

In these equations $\vec{\sigma}_\Delta(i)$ ($\vec{\tau}_\Delta(i)$) denotes the Δ isobar spin (isospin) operator, m_α the meson mass and Λ_α is a regularization cut-off mass. Besides, $Y(x)$ and $H(x)$ are given by

$$Y(x) = \frac{e^{-x}}{x} \qquad H(x) = \left(1 + \frac{3}{x} + \frac{3}{x^2}\right)Y(x) \tag{6}$$

while

$$S_{\Delta\Delta}(1,2) = 3\vec{\sigma}_\Delta(1)\cdot\hat{r}\vec{\sigma}_\Delta(2)\cdot\hat{r} - \vec{\sigma}_\Delta(1)\cdot\vec{\sigma}_\Delta(2). \tag{7}$$

The parameters of the model are quoted in table 1 and are taken from [11]. All given coupling constants refer to the coupling with nucleons. The couplings newly required for the $\Delta\Delta$ potential are scaled according to quark counting rules [12]. The rescaling factor is $\frac{1}{25}$ for contributions depending on spin and isospin and is 1 for all others. The pseudovector coupling constant f_π is related to the pseudoscalar coupling constant g_π through

$$f_\pi = g_\pi\frac{m_\pi}{2m_N} \tag{8}$$

where m_N is the mass of the nucleon. For the vector mesons, ρ and ω, the relation is given by,

$$f_\alpha = g_\alpha(1+k)\frac{m_\alpha}{2m_N} \tag{9}$$

where k is the ratio of the tensor-to-vector coupling given in table 1. Therefore, for the ω meson we have

$$f_\omega = g_\omega\frac{m_\omega}{2m_N} = \sqrt{3.48} \tag{10}$$

Table 1. Meson-exchange model parameters. All given coupling constants refer to the coupling with nucleons. The change in coupling constants from the purely nucleonic one-boson exchange potential to the $\Delta\Delta$ potential is given explicitly in the listed potentials. k gives the ratio of the tensor-to-vector coupling for the vector mesons ρ and ω.

Meson	$g_\alpha^2/4\pi$	k	m_α (fm^{-1})	Λ_α (fm^{-1})
π	14.4		0.7	4.2
σ	5.7		2.79	4.2
ρ	0.55	6.6	3.85	4.2
ω	20.0	0	3.97	4.2

Table 2. Binding energies B_2 (in MeV) of the $\Delta\Delta$ states with total angular momentum j and isospin i obtained in the chiral quark cluster model using only the direct term or the direct plus exchange terms of the interaction and in the meson-exchange model.

	B_2		
(j, i)	Quark direct	Quark direct + exchange	Meson exchange
(0, 1)	188.8	108.4	2035.3
(0, 3)	6.0	0.4	Unbound
(1, 0)	193.9	138.5	2651.7
(1, 2)	70.0	5.7	Unbound
(2, 1)	76.4	30.5	43.0
(2, 3)	35.6	Unbound	Unbound
(3, 0)	17.4	29.9	8.2
(3, 2)	30.7	Unbound	Unbound

and for the ρ meson,

$$f_\rho = g_\rho(1 + 6.6)\frac{m_\rho}{2m_N} = \sqrt{5.2}. \tag{11}$$

We have chosen a form factor with a cut-off mass $\Lambda_\alpha = 840$ MeV c^{-1}. This cut-off parameter has been taken as $\Lambda_\alpha = 1200$ MeV c^{-1} in [10, 11] which is comparable to the cut-offs used in the Bonn potential [13]. We note, however, that the form factor of the Bonn potential is of the dipole form $(\Lambda_\alpha^2 - m^2)^2/(\Lambda_\alpha^2 + p^2)^2$ (i.e. a monopole for each meson–nucleon–nucleon vertex), while the potential given by equations (1)–(5) has a form factor of the monopole form $(\Lambda_\alpha^2 - m^2)/(\Lambda_\alpha^2 + p^2)$ (i.e. a square-root monopole for each meson–nucleon–nucleon vertex). Thus, in order to have a similar fall-off at low momentum one must use $\Lambda_{\mathrm{monopole}} = \Lambda_{\mathrm{dipole}}/\sqrt{2}$ from which we obtained our value $\Lambda_\alpha = 840$ MeV c^{-1}.

The method of solution of the bound-state problem for the $\Delta\Delta$ and $\Delta\Delta\Delta$ systems has been described in [1]. In the case of the three-body system we calculated the binding-energy spectrum (that is, the energy of the states measured with respect to the three-body threshold) as well as the separation-energy spectrum (that is, the energy of the states measured with respect to the threshold of one free particle and a bound state of the other two). The deepest bound three-body state is not the one with the largest binding energy but the one with the largest separation energy since that state is the one that requires more energy in order to become unbound (that is, to move it from the bound state to the nearest threshold).

We show in tables 2 and 3 the binding energies of two and three Δs obtained from the chiral quark cluster model without Pauli effects (second column), from the chiral quark cluster model with Pauli effects (third column, results given in [1]), and from the meson-exchange model (fourth column).

Table 3. Binding energies B_3 (in MeV) of the $\Delta\Delta\Delta$ states with total angular momentum J and isospin I obtained in the chiral quark cluster model using only the direct term or the direct plus exchange terms of the interaction and in the meson-exchange model. We also give in parentheses the separation energies $B_3 - B_2$ (in MeV).

	$B_3(B_3 - B_2)$		
(J, I)	Quark direct	Quark direct + exchange	Meson exchange
$(\frac{1}{2}, \frac{1}{2})$	308.9(232.5)	84.0(53.5)	53.7(10.7)
$(\frac{1}{2}, \frac{3}{2})$	412.8(218.9)	139.2(0.7)	Unbound
$(\frac{1}{2}, \frac{5}{2})$	264.4(188.0)	Unbound	Unbound
$(\frac{1}{2}, \frac{7}{2})$	227.2(157.2)	6.3(0.6)	Unbound
$(\frac{1}{2}, \frac{9}{2})$	Unbound	Unbound	Unbound
$(\frac{3}{2}, \frac{1}{2})$	Unbound	109.5(1.1)	2038.7(3.4)
$(\frac{3}{2}, \frac{3}{2})$	406.2(212.3)	Unbound	Unbound
$(\frac{3}{2}, \frac{5}{2})$	318.6(129.8)	Unbound	Unbound
$(\frac{3}{2}, \frac{7}{2})$	101.3(31.3)	Unbound	Unbound
$(\frac{3}{2}, \frac{9}{2})$	127.6(92.0)	Unbound	Unbound
$(\frac{5}{2}, \frac{1}{2})$	265.4(189.0)	39.1(8.6)	47.0(4.0)
$(\frac{5}{2}, \frac{3}{2})$	320.2(126.3)	Unbound	Unbound
$(\frac{5}{2}, \frac{5}{2})$	223.1(146.7)	Unbound	Unbound
$(\frac{5}{2}, \frac{7}{2})$	187.9(117.9)	Unbound	Unbound
$(\frac{5}{2}, \frac{9}{2})$	Unbound	Unbound	Unbound
$(\frac{7}{2}, \frac{1}{2})$	229.2(152.8)	31.7(1.2)	44.0(1.0)
$(\frac{7}{2}, \frac{3}{2})$	117.3(40.9)	35.1(4.6)	Unbound
$(\frac{7}{2}, \frac{5}{2})$	188.9(112.5)	Unbound	Unbound
$(\frac{7}{2}, \frac{7}{2})$	155.0(119.4)	Unbound	Unbound
$(\frac{7}{2}, \frac{9}{2})$	Unbound	Unbound	Unbound
$(\frac{9}{2}, \frac{1}{2})$	Unbound	Unbound	Unbound
$(\frac{9}{2}, \frac{3}{2})$	124.8(94.1)	Unbound	Unbound
$(\frac{9}{2}, \frac{5}{2})$	Unbound	Unbound	Unbound
$(\frac{9}{2}, \frac{7}{2})$	Unbound	Unbound	Unbound

A general trend of the spectra of two and three Δs is that the channels with lower isospin, 0 and 1 for the two-body problem and $\frac{1}{2}$ for the three-body problem, are those favoured to have bound states. For the two-body problem, the quark model allows the existence of two bound states with isospin higher than 1, however, the corresponding binding energies are much smaller. In the case of the three-body problem, the quark model allows two bound states with isospin $\frac{3}{2}$ and one with isospin $\frac{7}{2}$. However, the $(\frac{1}{2}, \frac{3}{2})$ and $(\frac{1}{2}, \frac{7}{2})$ bound states are barely bound, i.e. they have separation energies smaller than 1 MeV. The state $(\frac{7}{2}, \frac{3}{2})$ has an anomalously large separation energy due to an accidental coincidence as explained in [1]. The preference for the lower isospin becomes evident by looking at the third and fourth columns of tables 2 and 3.

Let us first consider the results of the chiral quark cluster model. The consequences of quark antisymmetry can be easily isolated. Any baryon–baryon interaction obtained from the quark model consists of contributions with and without quark exchange; in other words, with and without Pauli effects at the level of quarks. While those contributions without quark exchange are long ranged, the quark exchange diagrams govern the short-range part of the interaction constituting the most significant difference with respect to meson-exchange models. To demonstrate the effect of quark antisymmetry, we have calculated the binding energies of the $\Delta\Delta$ and $\Delta\Delta\Delta$ systems of tables 2 and 3 using both the full interaction and the interaction with only the direct term, i.e. without exchange of quarks.

As shown in table 2, the $\Delta\Delta$ system has much more attraction with only the direct term than with the full model. One should emphasize the disappearance of the bound state in the case of the $(j, i) = (2, 3)$ and $(3, 2)$ channels which become strongly repulsive for distances below 1 fm due to the presence of quark Pauli blocking. These effects translate into an even more drastic reduction for the binding energies of the $\Delta\Delta\Delta$ system as shown in table 3. There, most of the bound states disappear upon inclusion of the exchange term of the $\Delta\Delta$ interaction and those that remain have rather small separation energies. The different effect of the exchange term between the two- and three-body cases is that the strong repulsive two-body channels $(j, i) = (2, 3)$ or $(3, 2)$ contribute in more than one three-body channel so that their effects become amplified in going from the two- to the three-body system.

In table 2 we show the results predicted by the quark model when only the direct term is considered and when quark exchange (the full potential) is included. As can be seen, the general trend of the quark-exchange diagrams is to decrease the attraction. Quark-exchange diagrams have the opposite sign to the direct potential for the OPE. In the case of the OGE there are only quark-exchange diagrams because the direct term always vanishes. The $(2, 3)$ and $(3, 2)$ channels present a strong Pauli blocking when the exchange diagrams are considered. For the $(0, 3)$, $(1, 2)$ and $(2, 1)$ channels the OGE is repulsive enough to be responsible for the decrease in binding. For the $(0, 1)$ and $(1, 0)$ channels the OGE is attractive, but the direct term of the OPE is also very attractive. When quark-exchange diagrams are connected, the attraction on the OPE is reduced more strongly than the attraction generated by the OGE, i.e. reducing the binding. A special case is found in the $(3, 0)$ channel, where the OGE and the direct term of the OPE are repulsive. Quark-exchange terms decrease the repulsion on the OPE more strongly than that generated by the OGE, giving the opposite effect to the $(0, 1)$ and $(1, 0)$ channels, in this case increasing the binding.

Let us consider next the results of the meson-exchange model. In this case there are only four $\Delta\Delta$ bound states which appear in order of decreasing binding energy in the channels $(j, i) = (1, 0), (0, 1), (2, 1)$ and $(3, 0)$. This model also gives rise to four $\Delta\Delta\Delta$ bound states which appear in order of decreasing separation energy in the channels $(J, I) = (\frac{1}{2}, \frac{1}{2}), (\frac{5}{2}, \frac{1}{2}), (\frac{3}{2}, \frac{1}{2})$ and $(\frac{7}{2}, \frac{1}{2})$. A very unpleasant feature of the meson-exchange results is that the binding energies in some of the channels are very large (~ 2000 MeV) and therefore unphysical; however, since the purpose of this comparison is mainly concerned with the ordering of the different channels, we do not worry too much about the absolute value of the energies. The extremely large energies found in some channels for the meson-exchange results have their origin in the regularization procedure followed in [10, 11]. The factor $(\Lambda/m)^3$ in the spin–spin contributions make them dominant and very large at short distances. One could have chosen a different regularization scheme such as $(\Lambda/m)^1$ and then the potential would become finite at short distances, but as we are interested in the level ordering of the states, we have preferred to leave the potential as in the original reference.

For the $\Delta\Delta$ case, the predicted bounds states, $(j, i) = (1, 0)$, $(0, 1)$, $(2, 1)$ and $(3, 0)$ also appear in the case of the NN system. In all three models of table 2, we find that the deepest bound state is $(j, i) = (1, 0)$, the second is $(j, i) = (0, 1)$, the third is $(j, i) = (2, 1)$ and the fourth is $(j, i) = (3, 0)$. The first three states also appear, and precisely in the same order, in the

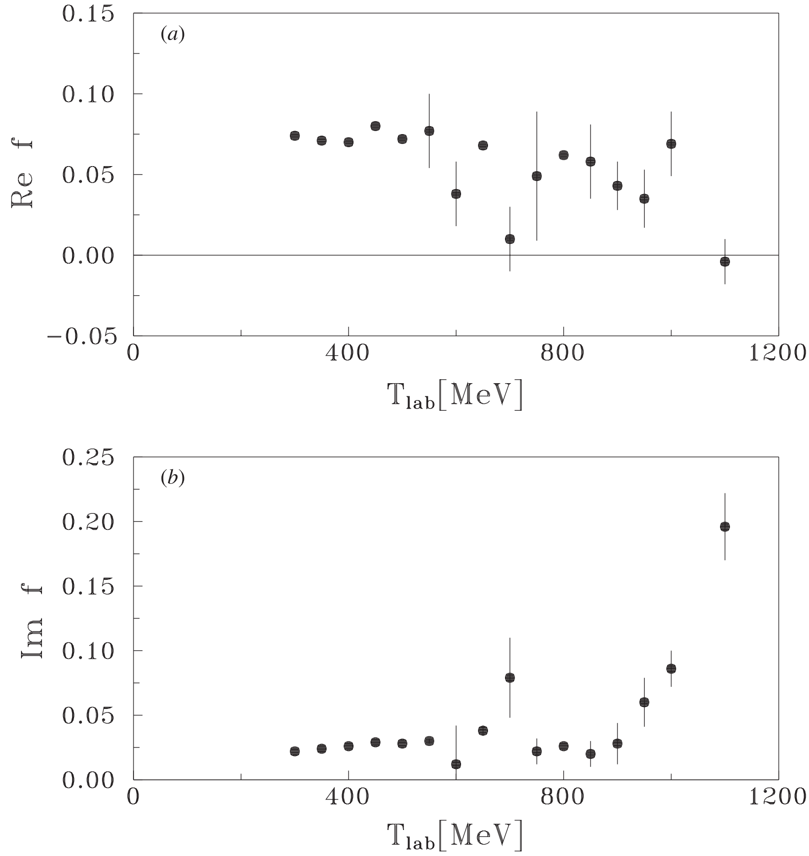

Figure 1. (*a*) Real and (*b*) imaginary parts of the single-energy solutions for the $^3D_3$ NN partial wave taken from [15].

case of the NN system. The $(j, i) = (1, 0)$ state is of course the deuteron, the $(j, i) = (0, 1)$ state is the $^1S_0$ virtual bound state, and the $(j, i) = (2, 1)$ state is the $^1D_2$ resonance that lies at ~ 2.17 GeV [14] (note that the $^3F_3$ NN resonance has no counterpart in table 2 because we calculate only even-parity states and $^3F_3$ has odd parity). Thus, the $(j, i) = (3, 0)$ state which is also allowed in the case of the NN system would correspond to a new nucleon–nucleon resonance that is predicted by our model.

It is interesting that some indication of a (3, 0) resonance can already be seen in the most recent analyses of the NN data by Arndt *et al* [15]. The $(j, i) = (3, 0)$ channel corresponds in the case of the nucleon–nucleon system to the $^3D_3$ partial wave. The most distinctive feature of a resonance is that as the energy increases the real part of the amplitude changes sign going from positive to negative, while the imaginary part becomes large, so that the amplitude describes a counterclockwise loop in the Argand diagram. The energy at which this change of sign occurs corresponds to the mass of the resonance. We show in figure 1 the real and imaginary parts of the $^3D_3$ amplitude obtained from the single-energy analysis of [15]. As one can see from this figure a resonance-like behaviour seems to be present at approximately 700 and 1100 MeV. These kinetic energies corresponds to invariant masses of 2.2 and 2.37 GeV so that in either case the ordering of the state agrees with that predicted by table 2. As mentioned before, the $\Delta\Delta$ bound state in the channel $(j, i) = (3, 0)$ has also been predicted by other models [4–8], and a method to search experimentally for this state has recently been proposed [9].

For the $\Delta\Delta\Delta$ case, the deepest bound state (that is the one with the largest separation energy) in all three models of table 3 is $(J, I) = (\frac{1}{2}, \frac{1}{2})$. This is also the case in the NNN system since there $(J, I) = (\frac{1}{2}, \frac{1}{2})$ is of course the triton, being practically the only bound state.

In summary, we have reanalysed the $\Delta\Delta$ and $\Delta\Delta\Delta$ systems making use of two different models: a quark-based model and a meson-exchange model. A similar ordering of states is obtained within both models. The main result of our study is the prediction of a new NN resonance with isospin 0, spin 3 and positive parity. A possible signal of this resonance appears in recent analyses of the NN data up to 3 GeV by Arndt *et al* [15]. Note that the theoretically predicted $\Delta\Delta\Delta$ bound state may also appear in nature as three-nucleon resonances.

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